

SOLVING THE 3D BIN PACKING PROBLEM TO IMPROVE TRANSPORT EFFICIENCY

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Abstract The application of geometric modelling in order to improve the efficiency of utilization of cargo space of vehicles is one of the steps forward in the practical application of the synthesis of basic geometric principles and computer programming. The paper presents the concept of solving a three-dimensional packing problem – the 3D Bin Packing Problem. The final result of the implementation of the aforementioned concept is the formation of a three-dimensional model of the packing plan, on the basis of which the compactness of cargo and the maximum utilization of cargo space is achieved. The applicability is shown in the example of packing packages within a company that deals with the transfer of postal express items. In such systems, solving the 3D Bin Packing Problem produces significant results. The reason for this is the stochastic nature of the transport requirements, which is primarily reflected in the different physical characteristics of the packages being transported. The efficient utilization of cargo space within transport systems contributes to the improvement of the realization of business activities and the reduction of costs

Keywords: Packing problems; 3D CAD model; cargo space engineering; transportation; postal items.

1. INTRODUCTION

Modern transportation systems represent a significant link in the goods distribution chain. They are faced with numerous challenges which often deviate from certain standardization criteria. This primarily refers to deadlines and the structure of the transported goods. Depending on their size and business policy, companies can use their own distribution systems or the systems hired from a third party. As a consequence, distribution chains and transportation networks of different complexity are created.

The postal network is one of the most developed logistics networks which daily transports a large number of consignments from one location to another. The realization of this process requires expenses in terms of money, energy, time, as well as other necessary resources. On the other hand, all companies believe that the successful business operation is reflected in the realized profit. It can be thus concluded that the efficient performance of activities and the responsible and justified use of resources are the pre-conditions for the successful operation of every company. Since transportation activities represent the most challenging segment of consignment shipment, their optimization is a significant factor of success. Dispatching and packing of consignments represent the reloading activities which significantly affect the efficiency of the transportation process. The manner of packing primarily affects the consignment safety and the cargo space usage, and consequently the efficiency of the total fleet. The significance of this problem is seen in the fact that in the future the postal system will witness an increasing number of consignments of non-standardized characteristics [1]. This paper analyzed the technological process of shipping express consignments in the postal

company. This company represents one of the largest logistics and distribution systems in Serbia and the region. Specific problems and challenges were identified in the process. They occur mostly due to the increasing number of non-standardized consignments in the system. One of the main causes of this tendency is the expansion of e-commerce chains and the share of postal companies in these systems, primarily in the segment of delivering the purchased goods. One of the selected problems is related to the inefficient usage of the cargo space of transportation vehicles due to the impossibility of compact packing of non-standardized consignments. This problem leads to the situation in which all consignments that are supposed to be transported to a delivery unit of the postal network cannot be packed in the envisaged cargo space of the delivery vehicle. This can result in delayed deliveries (waiting for a vehicle to return from the fieldwork or waiting for the next transportation line) or the necessity of hiring additional resources (means of transportation and couriers), which increases the expenses. This negatively impacts the sustainable development of postal services.

In the literature, this problem is named the 3D Bin Packing Problem, or the three-dimensional packing problem. It can also have other labels containing the following keywords - Container Loading Problems – CLP [2]. This problem can be solved by means of various suitable algorithms and realization software [1]. In order to optimize the use of the cargo space when transporting consignments, this paper proposed the introduction and application of a solving concept for the 3D Bin Packing Problem. On the basis of this concept, a geometric model of the packing plan was created in accordance with appropriate constraints. The results of the concept application were presented using an example from a real system, and the specialized software EasyCargo was used for solving the task of three-dimensional packing.

2. THE CONCEPT OF THE PROBLEM OF PACKING BINS INTO THE CARGO SPACE

In the literature, the packing problem is most often called the 3D Bin Packing Problem, i.e. the three-dimensional packing problem. However, this concept was preceded by the approaches for solving one-dimensional or two-dimensional problems. One-dimensional packing is the task of placing specific goods, objects or parcels into the corresponding cargo spaces by maintaining a high level of the cargo space usage and taking care of only one constraint – dimension (for example, the transportation vehicle capacity) [3]. The two-dimensional packing problem (2D Bin Packing Problem) is the improvement of the one-dimensional problem in terms of constraints which are considered while packing. Namely, this task considers two constraints – dimensions (for example, the parcel's length and width). One of the first tasks from this field found in the literature is packing (placing) the highest possible number of small rectangles of certain dimensions (without overlapping) on the surface of a larger rectangle [4]. The three-dimensional packing is a special case or a synthesis of the previously defined problems which takes into consideration the minimum of three constraints – dimensions [5]. The solution to this problem is creating a geometric model of the packing plan in the defined cargo space, container, on a pallet, etc. [1].

The 3D Bin Packing Problem is frequently associated with loading goods and pallets into the cargo space of a delivery vehicle or into a container, but it is also related to numerous other modifications, such as pallet packing. One of the first papers on this subject was written by Wright. It offered a solution related to the most suitable configuration for loading pallets into the cargo space [6]. The 3D

packing task can have different constraints and specific characteristics, such as rotating the parcels (yes/no), defining their priority when packing, stability analyses, etc. [7,8,9,10]. Lin et al. partially analyzed the concept of the 3D Bin Packing Problem in the field of parcel consignment delivery. In this study, the aim was to apply the 3D packing concept while eliminating reloading of parcels after each completed delivery [11]. When it comes to the consignment delivery and postal traffic, the problem was solved using the real system of the postal company in Serbia [1].

A significant number of algorithms for solving the tasks of 2D and 3D packing have been developed so far [12,13,14,15,16,17]. Some of them have been applied in the corresponding specialized software.

The general mathematical problem includes the task of packing parcels of specific dimensions into a cargo space of the pre-defined structure and capacity with the aim of its maximum exploitation, i.e. compact packing and reduction of space between the parcels [18]. In order to realize the mathematical model, one should know the dimensions of the available cargo spaces, as well as the dimensions of each bin which is to be loaded. One of the important suggestions is to pack the bins in so that their edges are parallel and perpendicular to the edges of the cargo space. A general model [18] is presented in the following text. The following variables and parameters were used:

- N – the total number of bins which should be packed;
- t – the total number of available cargo spaces;
- B – an arbitrarily large number;
- u_{ij} – a binary variable defining whether the bin i is placed into the cargo space j ($u_{ij} = 1$, when the bin i is placed into the cargo space j ; $u_{ij} = 0$, in other cases);
- o_j – a binary variable showing whether the cargo space j is occupied ($o_j = 1$, when the cargo space j is occupied; $o_j = 0$, in other cases);
- D_j, S_j, V_j – parameters related to the length, width and height of the cargo space j ;
- p_i, q_i, r_i – parameters related to the length, width and height of the bin i ;
- x_i, y_i, z_i – shows the location, or the coordinates of the front bottom left corner of the bin i in the cargo space;
- d_{xi}, d_{yi}, d_{zi} – binary variables indicating whether the length of the bin i is parallel to the axes X, Y and Z. If, for example $d_{yi} = 1$, it means that the length p is parallel to the axis Y, while $d_{yi} = 0$ is the opposite;
- s_{xi}, s_{yi}, s_{zi} – binary variables indicating whether the width of the bin i is parallel to the axes X, Y and Z. If, for example $s_{yi} = 1$, it means that the width q is parallel to the axis Y, while $s_{yi} = 0$ is the opposite;
- v_{xi}, v_{yi}, v_{zi} – binary variables indicating whether the height of the bin i is parallel to the axes X, Y and Z. If, for example $v_{yi} = 1$, it means that the height r is parallel to the axis Y, while $v_{yi} = 0$ is the opposite;

The following binary variables indicate the relative position between the bins:

- a_{ik} – has the value 1 if the bin i is to the left of the bin k ;
- b_{ik} – has the value 1 if the bin i is to the right of the bin k ;

- c_{ik} – has the value 1 if the bin i is behind the bin k ;
- g_{ik} – has the value 1 if the bin i is in front of the bin k ;
- e_{ik} – has the value 1 if the bin i is under the bin k ;
- f_{ik} – has the value 1 if the bin i is above the bin k .

When it comes to the observed dimensions, length always refers to the longest dimension, height to the shortest, while width refers to the medium dimension. Figure 1 represents an arbitrary cargo space j with two bins i and k . The fact that the bins i and k are in the cargo space j makes the binary variables u_{ij} and u_{kj} have the value 1. The analysis of the relative position of the bins shows that the bin i is to the right of and behind the bin k , so the variables b_{ik} and c_{ik} have the value 1. The values of the variables a_{ik} , g_{ik} , e_{ik} and f_{ik} amount to 0 in this case.

The analysis of the location of the bin i in relation to the cargo space j shows that the length of the bin p_i is parallel to the axis X ($d_{xi}=1; d_{yi}, d_{zi}=0$), the width of the bin q_i is parallel to the axis Z ($s_{zi}=1; s_{xi}, s_{yi}=0$), while the height of the bin r_i is parallel to the axis Y ($v_{yi}=1; v_{xi}, v_{zi}=0$). The orientation of the bin k in relation to the cargo space j is the same as in the case of the bin i , so the variables have the following values:

$$\begin{aligned} d_{xk} &= 1; d_{yk}, d_{zk} = 0; \\ s_{zk} &= 1; s_{xk}, s_{yk} = 0; \\ v_{yk} &= 1; v_{xk}, v_{zk} = 0. \end{aligned}$$

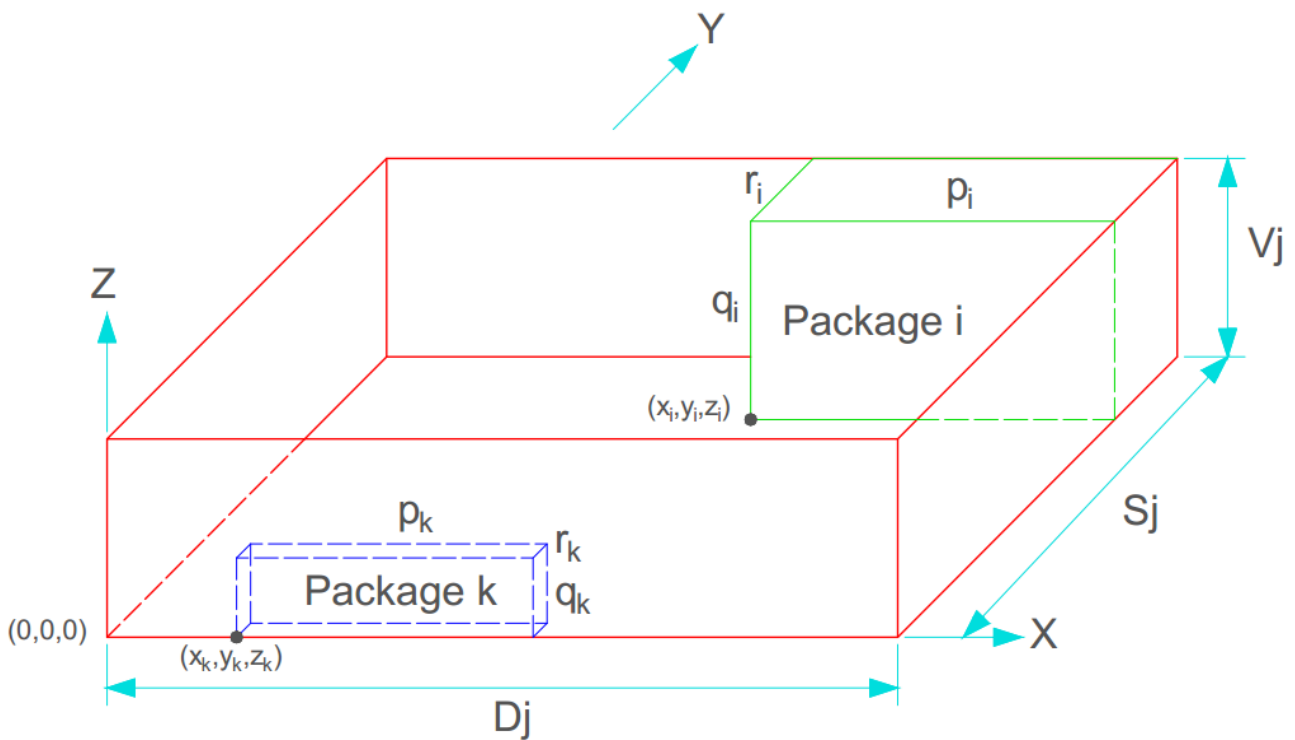


Figure 1. The cargo space j with the two packed bins (package) i and k .

The problem of packing bins into the cargo space is presented by means of the model of mixed integer linear programming [18]:

$$\min \sum_{j=1}^t D_j * S_j * V_j * o_j - \sum_{i=1}^N p_i * q_i * r_i \quad (1)$$

s.t.

$$x_i + p_i * d_{xi} + q_i * s_{xi} + r_i * v_{xi} \leq x_k + (1 - a_{ik}) * B, \quad \text{for all } i, k, i < k \quad (1)$$

$$x_k + p_k * d_{xk} + q_k * s_{xk} + r_k * v_{xk} \leq x_i + (1 - b_{ik}) * B, \quad \text{for all } i, k, i < k \quad (2)$$

$$y_i + q_i * s_{yi} + p_i * d_{yi} + r_i * v_{yi} \leq y_k + (1 - c_{ik}) * B, \quad \text{for all } i, k, i < k \quad (3)$$

$$y_k + q_k * s_{yk} + p_k * d_{yk} + r_k * v_{yk} \leq y_i + (1 - g_{ik}) * B, \quad \text{for all } i, k, i < k \quad (4)$$

$$z_i + r_i * v_{zi} + q_i * s_{zi} + p_i * d_{zi} \leq z_k + (1 - e_{ik}) * B, \quad \text{for all } i, k, i < k \quad (5)$$

$$z_k + r_k * v_{zk} + q_k * s_{zk} + p_k * d_{zk} \leq z_i + (1 - f_{ik}) * B, \quad \text{for all } i, k, i < k \quad (6)$$

$$a_{ik} + b_{ik} + c_{ik} + g_{ik} + e_{ik} + f_{ik} \geq u_{ij} + u_{kj} - 1, \quad \text{for all } i, k, j, i < k \quad (7)$$

$$\sum_{j=1}^t u_{ij} = 1, \quad \text{for all } i \quad (8)$$

$$\sum_{i=1}^N u_{ij} \leq B * o_j, \quad \text{for all } j \quad (9)$$

$$x_i + p_i * d_{xi} + q_i * s_{xi} + r_i * v_{xi} \leq D_j + (1 - u_{ij}) * B, \quad \text{for all } i, j \quad (10)$$

$$y_i + q_i * s_{yi} + p_i * d_{yi} + r_i * v_{yi} \leq S_j + (1 - u_{ij}) * B, \quad \text{for all } i, j \quad (11)$$

$$z_i + r_i * v_{zi} + q_i * s_{zi} + p_i * d_{zi} \leq V_j + (1 - u_{ij}) * B, \quad \text{for all } i, j \quad (12)$$

$$d_{xi}, d_{yi}, d_{zi}, s_{xi}, s_{yi}, s_{zi}, v_{xi}, v_{yi}, v_{zi}, a_{ik}, b_{ik}, c_{ik}, g_{ik}, e_{ik}, f_{ik}, u_{ij}, n_j = 0 \text{ or } 1,$$

$$x_i, y_i, z_i \geq 0$$

The objective function is used with the aim is to minimize the unused cargo space while packing. The constraints 1-6 ensure that there are no overlaps of the bin locations in the cargo space. Naturally, this constraint is valid only for the bins which are in the same cargo space, which is checked within the constraint 7. The constraint 8 guarantees that each bin can be only in one cargo space. If a bin is loaded into the cargo space, the cargo space is considered to be used (constraint 9). The constraints 10 – 12 ensure that all the packed bins are within the physical dimensions of the cargo space.

Therefore, it can be concluded that the following applies for the variables $d_{xi}, d_{yi}, d_{zi}, s_{xi}, s_{yi}, s_{zi}, v_{xi}, v_{yi}, v_{zi}$:

$$d_{xi} + d_{yi} + d_{zi} = 1;$$

$$s_{xi} + s_{yi} + s_{zi} = 1;$$

$$v_{xi} + v_{yi} + v_{zi} = 1;$$

$$d_{xi} + s_{xi} + v_{xi} = 1;$$

$$d_{yi} + s_{yi} + v_{yi} = 1;$$

$$d_{zi} + s_{zi} + v_{zi} = 1.$$

Having in mind the mentioned dependencies, the variables s_{xi} , s_{zi} , v_{xi} and v_{yi} can be removed from the model, i.e. they can be expressed using other variables. Following these alterations, the model will be as follows:

$$x_i + p_i * d_{xi} + q_i * (d_{zi} - s_{yi} + v_{zi}) + r_i * (1 - d_{xi} - d_{zi} + s_{yi} - v_{zi}) \leq x_k + (1 - a_{ik}) * B \quad (1a)$$

$$x_k + p_k * d_{xk} + q_k * (d_{zk} - s_{yk} + v_{zk}) + r_k * (1 - d_{xk} - d_{zk} + s_{yk} - v_{zk}) \leq x_i + (1 - b_{ik}) * B \quad (2a)$$

$$y_i + q_i * s_{yi} + p_i * (1 - d_{xi} - d_{zi}) + r_i * (d_{xi} + d_{zi} - s_{yi}) \leq y_k + (1 - c_{ik}) * B \quad (3a)$$

$$y_k + q_k * s_{yk} + p_k * (1 - d_{xk} - d_{zk}) + r_k * (d_{xk} + d_{zk} - s_{yk}) \leq y_i + (1 - g_{ik}) * B \quad (4a)$$

$$z_i + r_i * v_{zi} + q_i * (1 - d_{zi} - v_{zi}) + p_i * d_{zi} \leq z_k + (1 - e_{ik}) * B, \text{ for all } i, k, i < k \quad (5a)$$

$$z_k + r_k * v_{zk} + q_k * (1 - d_{zk} - v_{zk}) + p_k * d_{zk} \leq z_i + (1 - f_{ik}) * B, \text{ for all } i, k, i < k \quad (6a)$$

$$x_i + p_i * d_{xi} + q_i * (d_{zi} - s_{yi} + v_{zi}) + r_i * (1 - d_{xi} - d_{zi} + s_{yi} - v_{zi}) \leq D_j + (1 - u_{ij}) * B \quad (10a)$$

$$y_i + q_i * s_{yi} + p_i * (1 - d_{xi} - d_{zi}) + r_i * (d_{xi} + d_{zi} - s_{yi}) \leq S_j + (1 - u_{ij}) * B \quad (11a)$$

$$z_i + r_i * v_{zi} + q_i * (1 - d_{zi} - v_{zi}) + p_i * d_{zi} \leq V_j + (1 - u_{ij}) * B \quad (12a)$$

One of the additional constraints which can be included in the model, and which is particularly important for transport, is the allowed total weight of the bins, i.e. the cargo space capacity:

$$\sum_{j=1}^t \sum_{i=1}^N u_{ij} * m_i \leq M_j \quad (13)$$

where m_i is the weight of the consignment i , while M is the capacity of the observed cargo space.

The presented model is one of the general models dealing with the observed problem. These models have been developed over time. Depending on the requirements, various functionalities and constraints can be included, as in the example of defining the capacity constraint. The development of mathematical models and algorithms has considerably contributed to the development of software for solving the task of dimensional packing.

3. THE PROPOSED CONCEPT FOR SOLVING THE 3D BIN PACKING PROBLEM WHILE TRANSPORTING EXPRESS CONSIGNMENTS

The significance of solving the problem of three-dimensional packing while transporting express consignments relies on the fact that the transportation systems are faced with an increasing number of consignments with non-standardized dimensions. The sources of these consignments are mainly e-commerce chains. Namely, in order to improve their position in the market, attract a larger number of users and increase the profit, postal companies have to take part in the e-commerce chains. Consequently, they accept the transportation of an increasing number of non-standardized consignments. Non-standardized consignments are all the consignments whose dimensions and other features do not correspond to the regulated standards. Automated postage systems are adjusted to

operate with standardized consignments so it is impossible to process non-standardized consignments in an automated manner. Therefore, these consignments are mostly processed manually.

Significant problems arise when packing non-standardized consignments into the cargo space of a transportation vehicle. These consignments cannot be packed into the standard packaging for transporting express consignments, so they are loaded into the transportation vehicle in their own packaging (if they have one). This makes completely compact packing and minimizing the cargo space usage impossible, which can indirectly cause numerous problems. Due to the lack of space in the cargo space, it is sometimes impossible to pack all the consignments scheduled by the defined transportation timetable in accordance with the delivery deadlines. Two scenarios are possible in the mentioned situation: transportation of consignments by later departures (it can result in the delayed delivery of consignments); or engaging additional resources (transportation vehicles and couriers) in order to realize the transportation of the remaining consignments according to the pre-defined schedule. In the latter scenario, there are additional expenses which cannot be compensated through the service cost. When hiring an additional transportation vehicle, the negative impact on the environment is evident. Non-standardized consignments are subject to damages during the reloading and transportation processes. Namely, the impossibility of compact packing leaves a larger space between the consignments, which increases the possibility that the consignments will move, slide and squeeze during transportation. As mentioned above, the systems of the postal companies are faced with an increasing number of different consignments, i.e. goods of various features, sensitivity and value, which additionally aggravates the stated problem.

A concept of solving the 3D bin packing problem has been proposed. This concept implies creating a geometric model, i.e. a plan which should be the basis for conducting packing, depending on the defined characteristics of the cargo space and consignments. The benefits of the application of this approach lie in better exploitation of the cargo space, and are primarily perceived in eliminating delays, improving consignment safety and decreasing the number of required transportation resources (reducing the costs and negative impact on the environment).

The realization of the concept for solving the 3D bin packing problem within the system of transporting express consignments can be presented in the following steps [1]:

- Step 1: collecting information about the physical characteristics of consignments (length, width, height and weight);
- Step 2: defining the cargo space, i.e. the transportation vehicle;
- Step 3: the application of the software for solving the 3D bin packing problem;
- Step 4: the analysis of the obtained solution;
- Step 5: packing of the consignments according to the defined geometric model of the packing plan.

4. SOLVING THE 3D BIN PACKING PROBLEM USING AN EXAMPLE FROM THE REAL SYSTEM FOR TRANSPORTING EXPRESS CONSIGNMENTS

The analyzed packing task refers to an example from a real system of the postal company. The consignments were supposed to be transported from the processing centre to the corresponding unit

of the postal network. The vehicle Peugeot Partner was available. When loading the consignments into the transportation vehicle, certain number of them (approximately 10%-15% of the total quantity) could not fit into the envisaged cargo space. The application of the proposed concept enables creating the geometric model of the packing plan, which will help all consignments to be packed, and indicating the impossibility of loading all the consignments in the envisaged cargo space. The results related to the latter situation can initiate changes in transport organization (hiring a transportation vehicle with a larger cargo space capacity, distributing the consignments to two or more transportation vehicles).

The first step involves collecting the information about the physical characteristics of consignments. This concept is not implemented in the observed company (there is no system for collecting all necessary information). Thus, in the studied case it was not possible to precisely define the required characteristics because the standard business process does not include such records. Table 1 shows the requirements, i.e. the number and characteristics of the consignments which should be packed into the cargo space of a specific transportation vehicle. Precise data about the characteristics were considered for standard consignments (packaging dimensions), while the characteristics of the remaining non-standardized consignments were defined in cooperation with the company employees who worked on the jobs of consignment reloading and directing. Modern systems based on 3D scanners can collect precise information about the physical characteristics of consignments. Accurate data on the consignments' weights were not available, but the total weight was within the limits of the cargo load capacity of the envisaged transportation vehicle, according to the employees' estimation. This is the reason why the consignments were assigned arbitrary weight values.

Table 1. Characteristics of the requirements.

Packaging type	Dimensions (length x width x height) [mm]	Quantity
Cardboard bins T1	(250x175x100)	53
Cardboard bins T2	(350x250x120)	33
Bulk bins	(550x500x400)	9
Sacks	(1000x350x350)	2
Consignments of non-standardized packaging		47

Step 2 defines the cargo space where the consignments are supposed to be packed. In the analyzed case, the used vehicle was Peugeot Partner, with the capacity of approximately 650kg, and the following dimensions of the cargo space: length - 1800mm; width - 1620mm; height - 1250mm.

In step 3, the software is applied to solve the 3D Bin Packing Problem, i.e. the geometric model of the packing plan is created. In this specific case, the obtained solution stated that all the stored consignments could be packed in the defined cargo space. However, in the reality 10% - 15% of the total quantity of the consignments remained out of the cargo space. In the solution, the cargo space usage, presented by the occupancy rate, amounted to around 80%. The consignments requiring special handling, particularly the fragile ones, which were outside of the bulk bins, were included in the packing plan by the software users in order to increase their safety level [1]. Figures (Fig. 2 – 4) offer the geometric presentation of the packing plan models using different views, which was obtained as the graphic output of the software. Thus, obtained solution can be decomposed in order to obtain a more detailed visual plan. The decomposition level depends on the consignment structure,

i.e. the level of visibility of the consignments' position in the cargo space [1]. The following figures (Fig.5 – Fig.7) show the packing plan with an arbitrary decomposition level.

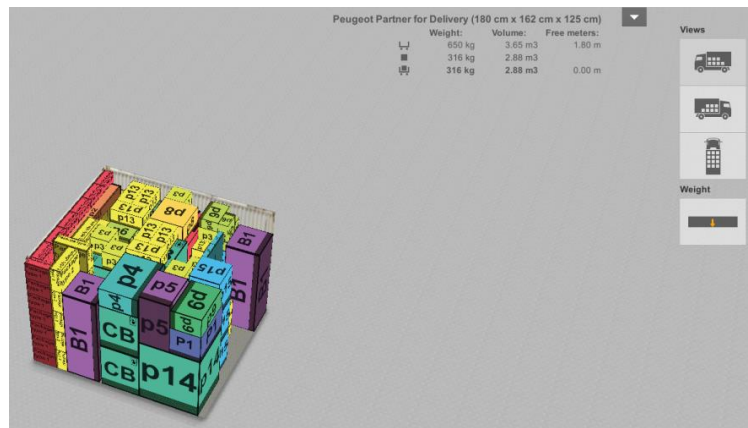


Figure 2. The geometric model of the packing plan (perspective view 1).

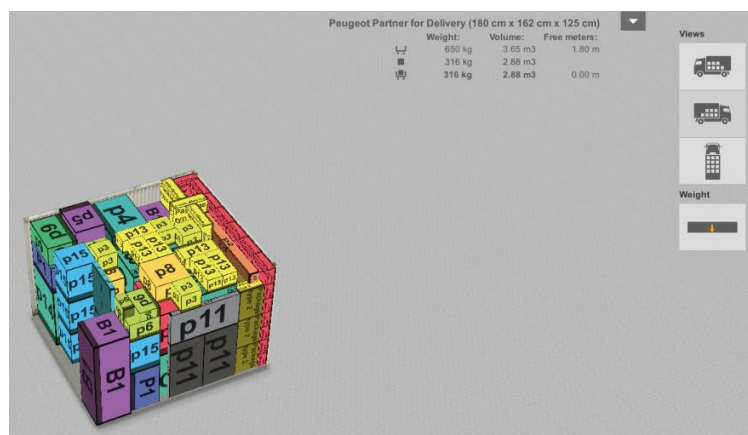


Figure 3. The geometric model of the packing plan (perspective view 2).

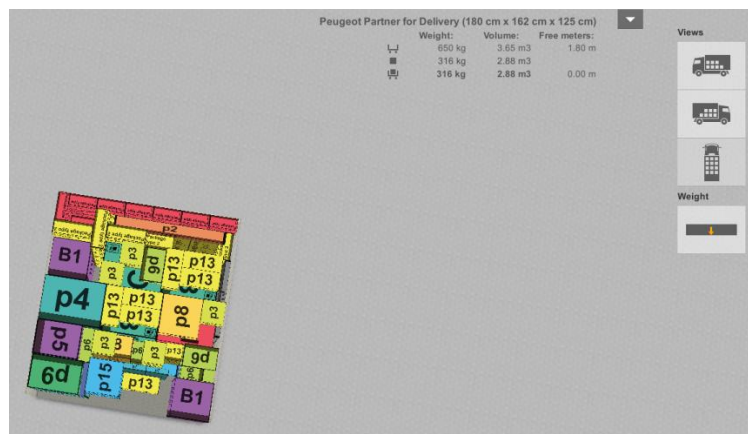


Figure 4. The geometric model of the packing plan (perspective view 3).

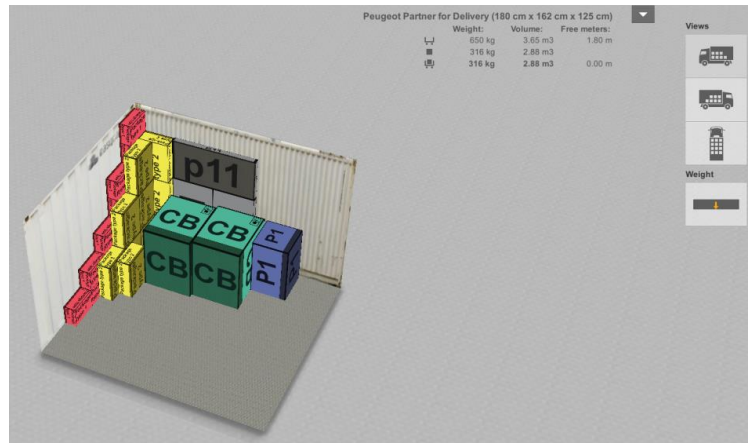


Figure 5. The decomposed geometric model of the packing plan – level 1.

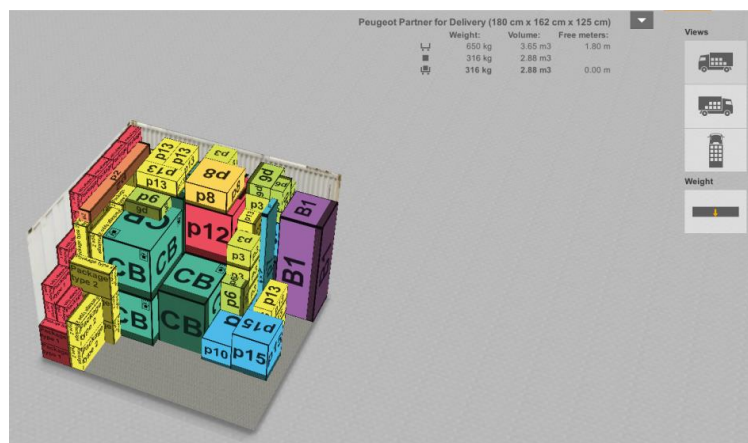


Figure 6. The decomposed geometric model of the packing plan – level 2.

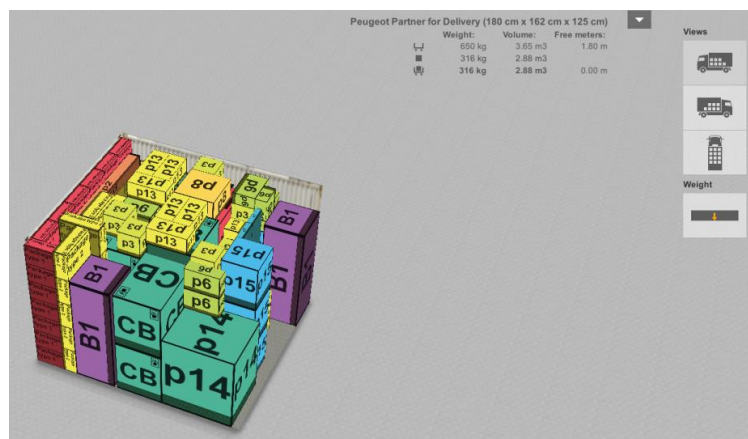


Figure 7. The decomposed geometric model of the packing plan – level 3.

The number of decomposition levels depends on the structure of the geometric model of the packing plan. In theory, the number of decomposition levels can be equal to the number of consignments which are to be packed. The last level represents the complete packing plan which is presented in this paper by means of various perspective views, as the obtained final solution.

The analysis of the solution involves checking whether all the consignments are in the packing plan, as well as checking the position of the consignments requiring special handling and including sensitive content. The software enables manual correction of the consignment position if required, as well as the generation of the records regarding their status. On the basis of the solution and created geometric model of the packing plan, transportation vehicles are analyzed in order to find out the most suitable one for transportation. In the case of the low level of the cargo space usage, a transportation vehicle of lower capacity should be considered. On the other hand, when the cargo space capacity of the transportation vehicle is insufficient for all consignments, the task to be solved is the selection of the transportation concept – a larger transportation vehicle or 2 or more small vehicles [1]. In the studied situation, there were no significant corrections, and the ones which were conducted are part of the presented final solution of the geometric model of the packing plan.

5. CONCLUSION

Improving the sustainability of transportation systems represents a significant initiative for transportation companies and for the companies whose operation is based on transportation activities, and for society as a whole. Distribution systems ensure the transportation of various goods to specific locations, while forming a complex logistics chain based on the developed and strong infrastructure. Depending on the nature of the distributed goods, the technological process of transportation has specific characteristics. One of the common tasks is the efficient usage of the available transportation resources. This study analyzed the problem of the efficient usage of the cargo space of transportation vehicles.

The paper observed the technological process of transporting express consignments in the postal company which represents one of the largest logistics and distribution centres in Serbia and the region. The observed problem was related to packing postal consignments into the cargo space of transportation vehicles while transporting the consignments between postal centres and delivery units of the postal network. In the future, the expansion of e-commerce will result in a larger number of non-standardized consignments, which will make the packing problem even more complex. In order to solve the mentioned problem, i.e. to optimize the cargo space usage when transporting consignments, the application of a concept for solving the packing problem (3D Bin Packing Problem), was proposed in the paper. On the basis of this concept, a geometric model of the packing plan was created according to corresponding constraints. The application of the concept can generate a solution for compact consignment packing with a high level of the cargo space usage or indicate the insufficient cargo space capacity of the analyzed transportation vehicle. In the literature and in practice, there have been a large number of algorithms and examples of software for solving the 3D Bin Packing Problem. Using an example from a real system, the paper proved the applicability of the proposed concept. The results of the application of the EasyCargo software were presented in the form of geometric models of the packing plan.

References

- [1] Lazarević, D., Dobrodolac, M. and Petrović, M., 2019. Optimizacija iskorišćenja tovarnog prostora formiranjem geometrijskog modela plana pakovanja pošiljaka. Proceedings of The International Scientific Conference on Information Technology and Data Related Research Sinteza 2019, Novi Sad, Serbia. pp 45-51.
- [2] Mladenović, S., Zdravković, S., Vesković, S., Janković, S., Đorđević, Ž., and Đalić, N., 2019. Development of a Novel Freight Railcar Load Planning and Monitoring System. *Symmetry*, 11(6). pp 756.
- [3] Johnson, D. S., Demers, A., Ullman, J. D., Garey, M. R. and Graham, R. L., 1974. Worst-case performance bounds for simple one-dimensional packing algorithms. *SIAM Journal on computing*, 3(4). pp 299-325.
- [4] Erdős, P. and Graham, R. L., 1975. On packing squares with equal squares. *Journal of Combinatorial Theory*, 19(1). pp 119-123.
- [5] Martello, S., Pisinger, D. and Vigo, D., 2000. The three-dimensional bin packing problem. *Operations research*, 48(2). pp 256-267.
- [6] Wright, P., 1974. Pallet loading configurations for optimal storage and shipping. *Paperboard and Packing*, pp 46-49.
- [7] Dube, E., Kanavathy, L. R., and Woodview, P., 2006. Optimizing Three-Dimensional Bin Packing Through Simulation. In Sixth IASTED International Conference Modelling, Simulation, and Optimization, Gaborone, Botswana.
- [8] Miyazawa, F. K. and Wakabayashi Y., 2009. Three-dimensional packings with rotations. *Computers & Operations Research*, 36(10). pp 2801-2815.
- [9] Junqueira, L., Morabito, R. and Yamashita, D.S., 2012. Three-dimensional container loading models with cargo stability and load bearing constraints. *Computers & Operations Research*, 39(1). pp 74-85.
- [10] Lim, A., Ma, H., Xu, J. and Zhang, X., 2012. An iterated construction approach with dynamic prioritization for solving the container loading problems. *Expert Systems with Applications*, 39(4). pp 4292-4305.
- [11] Lin, C. C., Kang, J. R., Liu, W. Y and Li, C. C., 2016. On two-door three-dimensional container packing problem under home delivery service. *Journal of Industrial and Production Engineering*, 33(3). pp 205-214.
- [12] Pisinger, D., 2002. Heuristics for the container loading problem. *European journal of operational research*, 141(2). pp 382-392.
- [13] Egeblad, J. and Pisinger, D., 2009. Heuristic approaches for the two-and three-dimensional knapsack packing problem. *Computers & Operations Research*, 36(4). pp 1026-1049.
- [14] Joung, Y. K., and Do Noh, S., 2014. Intelligent 3D packing using a grouping algorithm for automotive container engineering. *Journal of Computational Design and Engineering*, 1(2). pp 140-151.
- [15] Feng, X., Moon, I. and Shin, J., 2015. Hybrid genetic algorithms for the three-dimensional multiple container packing problem. *Flexible Services and Manufacturing Journal*, 27(2-3). pp 451-477.
- [16] Junqueira, L. and Morabito, R., 2015. Heuristic algorithms for a three-dimensional loading capacitated vehicle routing problem in a carrier. *Computers & Industrial Engineering*, 88. pp 110-130.
- [17] Paquay, C., Schyns, M. and Limbourg, S., 2016. A mixed integer programming formulation for the three-dimensional bin packing problem deriving from an air cargo application. *International Transactions in Operational Research*, 23(1-2). pp 187-213.

- [18] Chen, C. S., Lee, S. M. and Shen, Q. S., 1995. An analytical model for the container loading problem. *European Journal of Operational Research*, 80(1). pp 68-76.